On the Spatial Analysis of Shoaling Waves

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ABSTRACT

This study presents the application of the wavelet and the Empirical Mode Decompositions (EMD) analyses for the spatial evolution of shoaling waves. Both analyses transform shoaling wave data into wave energy spectrum as a function of wavenumber and distance to shoreline. The wave data used in the analysis are simulated linear and nonlinear waves as well as transect profiles from 3D ocean wave topography measured by an airborne topographic mapper (ATM). The wavelet analysis shows harmonic generation as wave shoals. The EMD analysis displays a strong intra-wave modulation of the peak wavenumber with its modulation amplitude increasing as waves move into shallower waters. For the in-situ spatial wave data, the dominant feature is the groupiness of wave energy.

1. Introduction

Recently, an airborne topographic mapper (ATM, an airborne scanning laser ranging system) has been deployed to acquire three-dimensional (3D) topography of ocean surface waves off the coast of Duck, North Carolina. As waves move into shallower waters, the measurements reveal a decreasing wavelength along with increasingly sharper crest and flatter trough. This shoaling process could change significantly within a short distance. In this study, we explore the application of two methods for the spatial analysis of shoaling waves. Both methods transform shoaling wave data into wave energy spectrum as a function of wavenumber, k, and space, x, (distance to shoreline). The first method is the Fourier-based Morlet wavelet analysis, which is essentially an adjustable-windowed Fourier spectral analysis (Huang et al. 1998). The Morlet wavelet has been applied to analyze nonstationary ocean waves in many studies. Details of the Morlet wavelet analysis can be found in Huang et al. (1998). The second method is the empirical mode decomposition method (EMD) combined with Hilbert spectral analysis developed by Huang et al. (1998) for analyzing nonlinear and nonstationary data. The EMD method decomposes data into a finite number of intrinsic mode functions (IMF). The Hilbert transform is applied to each IMF element to yield the wave spectrum. The major difference between the wavelet and the EMD method in analyzing wave data is that the wavelet analysis decomposes the data into spectral elements associated with the pre-determined FFT frequency (wavenumber) bands. The EMD, on the other hand, decomposes the data into IMFs, which have no pre-determined frequencies (wavenumber). Huang et al. (1998) state that the Fourier-based wavelet analysis interprets nonlinearity as extra harmonics while the EMD reveals inter- and intra-wave modulations of the nonlinear dynamics.

2. Data Analysis

The shoaling wave data used in this study are obtained from both numerical simulations and from in-situ ATM measurements at Duck, North Carolina. The shoaling waves are simulated with and without nonlinear terms by the models of Kaihatu and Kirby (1995). The simulated waves have a period of 12 s with a normal incident angle. For the in-situ data, the offshore wave field is dominated by a 10-sec easterly swell. Details of ATM data can be found in Hwang et al. (2000).

a. Analysis of linear shoaling waves

Figure 1a shows the simulated 1D shoaling surface waves and the bathymetry profile along a cross shore section of the linear wave field. Also shown are the images of spectra of the wavelet (Fig. 1b) and the EMD (Fig. 1c) analyses. The shoaling wave profile displays a gradually decreasing wavelength as water depth decreases. Most of the wave energy of the wavelet spectrum concentrate around the peak wavenumber, which increases from 0.05 to 0.1 rad/m as water depth decreases from 13 to 4 m. Very similar results are found in the EMD spectrum, which has a narrower energy spreading around the peak wavenumber as compared to the wavelet spectrum. The larger energy spreading in the wavelet spectrum can be attributed to an energy leakage in the wavenumber domain due to a less-than-ideal choice for the basis function of the wavelet (Zhu et al., 1997). The relation between the water depth and peak wavenumber, k_p, (the wavenumber associated with the maximum spectrum density at distance x) from the wavelet and EMD analyses is shown in Figure 2. The solid line represents the linear dispersion relationship. Peak wavenumbers from wavelet and EMD spectra are in good agreement with the linear dispersion relation, except at the very nearshore region ($h/L_o < 0.01$).

b. Analysis of nonlinear shoaling waves

Figure 3a shows 1D shoaling surface wave and the bathymetry profile along a cross shore section of the simulated nonlinear wave field. The wave profile displays a strong nonlinear effect with increasingly sharper crest and flatter trough as wave moves into shallower waters. The spectra of wavelet and EMD analyses are shown in Figs 3b and 3c, respectively. The wavelet spectrum shows a energy concentration in a narrow strip around the peak wavenumber (at $x >$ 1000m), which gradually increases as water depth decreases. At $x <$ 1000m, a secondary energy peak starts to appear. The secondary peak gradually increases in both frequency and energy and becomes the dominant peak as water depth further decreases. Also a third energy peak gradually appears and becomes dominant at very shallow water (x = 200 m). The second and the third peaks are the second and third harmonics induced by nonlinear effects. The energy variation of the EMD spectrum is very different from the wavelet spectrum. The EMD spectrum at $x >$ 1200 m displays very narrow energy spreading around the peak wavenumber, which modulates around 0.05 rad/m. The modulation is in phase with the surface wave variation with its amplitude increasing as water depth decreases. At $x <$ 1200 m, as water depth further decreases, the modulation decreases significantly and remains around 0.08 rad/m. A second modulation starts to appear at frequencies higher than 0.08 rad/m. This modulation is more discrete and has a lager modulation amplitude. Figure 4 shows the relation of water depth and the mean wavenumber, $\langle k \rangle = \int kE(k,x)dk / \int E(k,x)dk$, from the spectra of the wavelet and the EMD analyses. In general, both mean wavenumbers increase as water depth decreases, which is consistent with the linear dispersion relation shown as the solid curve. The mean wavenumber of the wavelet analysis displays an intra-wave modulation with its amplitude increasing as water depth decreases. Most of the mean wavenumbers of the wavelet analysis are higher than that calculated by linear dispersion relation. Very similar results are observed from the mean wavenumber of the EMD analysis, which has a more pronounced intra-wave modulation around the solid curve representing the linear dispersion relation.

c. Analysis of random shoaling waves

The in-situ wave profile is from a cross shore section of the 3D ocean surface wave topography acquired by the ATM. The sea state was dominated by a 10-sec easterly swell. Figure 5a shows the wave profile and bathymetry along the cross-shore section. The spectrum of the wavelet and the EMD analyses are shown in Figs. 5b and 5c, respectively. One significant feature of the in-situ data is the strong group structure in the wavelet and EMD spectra.. At very shallow water ($x <$ 400 m), there is a very strong second and third energy peaks in the wavelet spectrum similar to the analysis of the simulation data. The EMD spectrum shows the peak wavenumber modulations of its IMF. The relation between water depth and the mean wavenumber from the wavelet and the EMD analyses is shown in Figure 6. Even though the data are much more scattered as compared to those of the simulation data, the results of the wavelet and the EMD analyses are similar, especially at shallower water depths ($h/L_o <$ 0.04).

3. Summary

In this study, we explore the application of the wavelet and EMD analyses for the spatial evolution of shoaling waves in nearshore waters. Wave data are from numerical simulations based on linear and nonlinear shoaling wave models and from in-situ 3D surface topography acquired by the ATM. For the simulated linear shoaling waves, both analyses show that the energy concentrates in a very narrow strip around the peak wavenumber that follows the liner dispersion relation. For the analysis of simulated nonlinear waves, as water depth decreases, the spectra of the wavelet analysis shows second and the third energy peaks that are associated with the harmonics of the primary wavenumber. The harmonics become dominant as waves move into shallower waters. The relation between the water depth and the mean wavenumber from both wavelet and EMD analyses displays a strong intra-wave modulation with an increasing modulation amplitude as water depth decreases. For the analysis of in-situ spatial wave data, both wavelet and EMD analyses display very similar features as observed in the analysis of simulation data with an exception that the in-situ data have a strong group structure.

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REFERENCES

Huang, N.E. et al., 1998: The empirical mode decompositions and the Hilbert spectrum for nonlinear and non-stationary time series analysis. *Proc. R. Soc. London* **A454**, 903-995.

Hwang, P. A., D. W. Wang, E. J. Wash, W. B. Krabill and R. N. Swift, 2000: Airborne measurements of the wavenumber spectra of ocean surface waves. Part 1. Spectral slope and dimensionless spectral coefficient. *J. Phys. Oceanogr.* (in press)

Kaihatu, J. M., and J. T. Kirby, 1995: Nonlinear transformation of waves in finite water depth. *Phys. Fluids*, **7**, 1903-1914.

Zhu, X., Z. Shen, S. D. Eckermann, M. Bottner, I. Hirota, and J. Ye, 1997: Gravity wave characteristics in the middle atmosphere derived from the Empirical Mode Decomposition method. *J. Geophys. Res.*, **102**, 16545-16561.

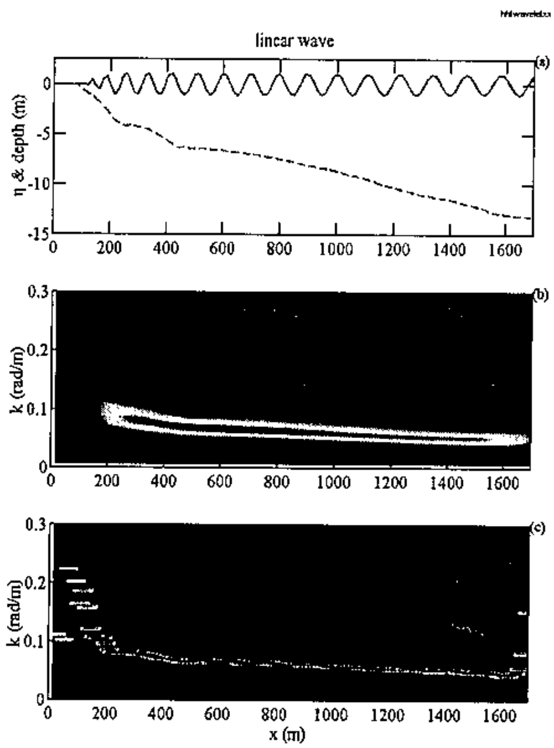

Fig. 1 (a) Linear shoaling wave profile and bathymetry, (b) spectrum of wavelet analysis, (c) spectrum of EMD analysis.

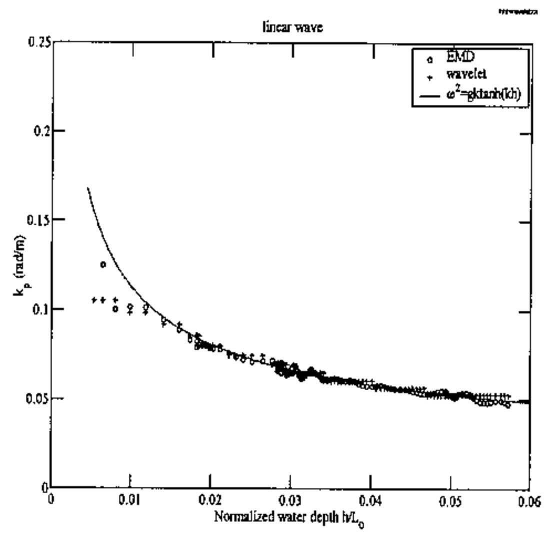

Fig. 2. Peak wavenumbers of the wavelet and the EMD analyses versus normalized water depth. L_o is deep water wavelength.

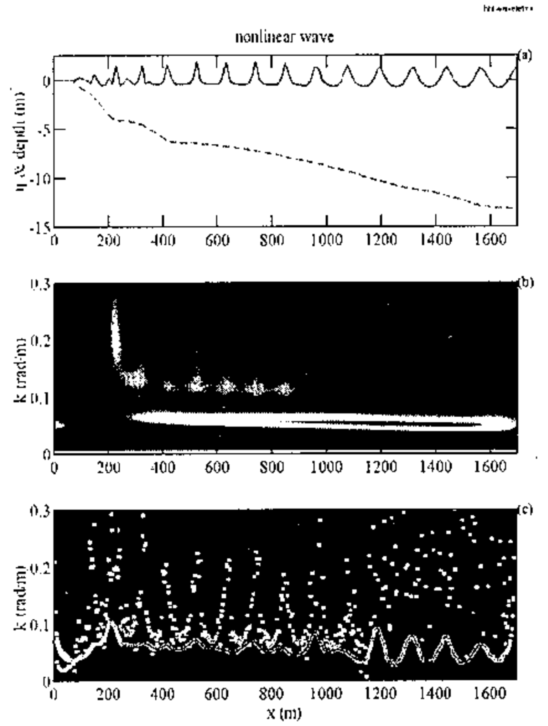

Fig. 3. (a) Nonlinear shoaling wave profile and bathymetry, (b) spectrum of wavelet analysis, (c) spectrum of EMD analysis.

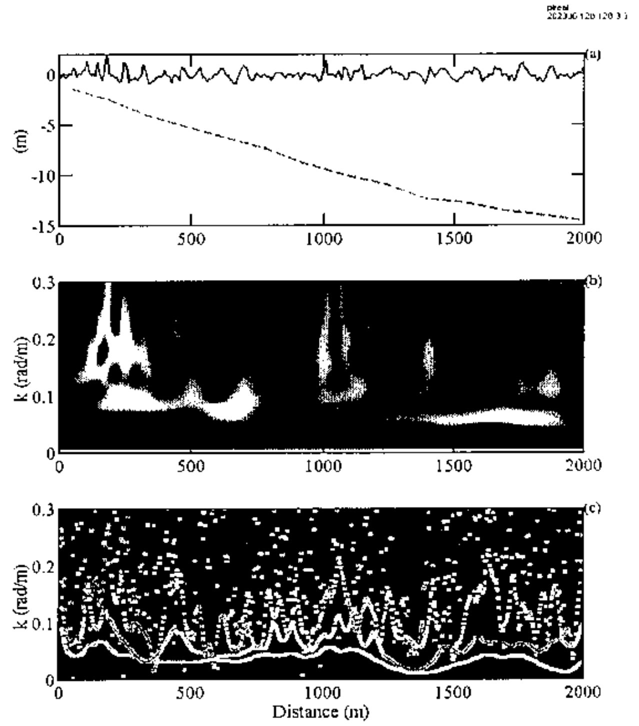

Fig.5. (a) In-situ ATM wave profile and bathymetry, (b) spectrum of wavelet analysis, (c) spectrum of EMD analysis.

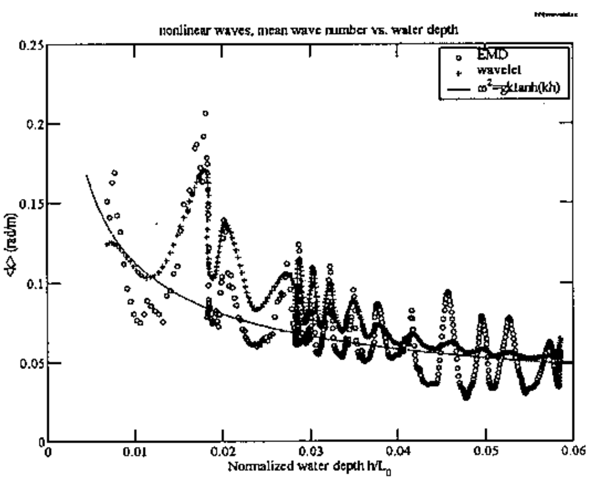

Fig. 4. Mean wavenumbers of wavelet and EMD analyses versus normalized water depth.

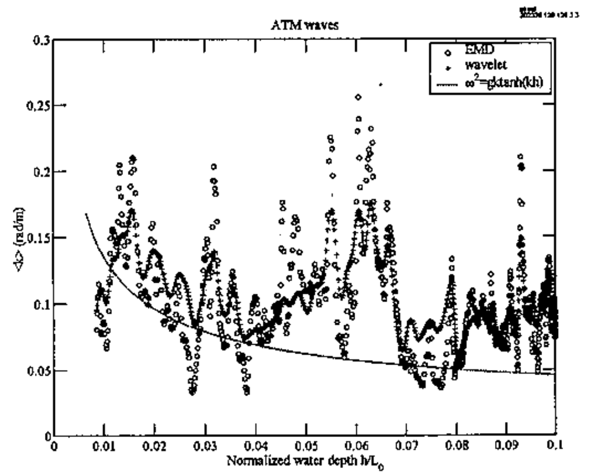

Fig. 6. Mean wavenumbers of wavelet and EMD analyses versus normalized water depth.

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